

The Journal of Fourier Analysis and Applications

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VOLUME 27 · NUMBER 1 · 2021

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Office of Publication

Birkhäuser Boston, 1 New York Plaza, Suite 4600, New York, NY 10004, USA. Telephone: 212-460-1500; email: journalsny@springer-sbm.com, website: springeronline.com, Production Editor: Bill Payne.



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Aim and Scope

The Journal of Fourier Analysis and Applications will publish results in Fourier analysis, as well as applicable mathematics having a significant Fourier analytic component. Appropriate manuscripts at the highest research level will be accepted for publication. Because of the extensive, intricate, and fundamental relationships between Fourier analysis and so many other subjects, select and readable surveys will also be published. These surveys will include historical articles, research tutorials, and expositions of specific topics.

The Journal of Fourier Analysis and Applications will provide a perspective and means for centralizing and disseminating new information from the vantage point of Fourier analysis. The breadth of Fourier analysis and diversity of its applicability require that each paper should contain a clear and motivated introduction, which is accessible to all of our readers.

Applications include the following topics:

Antenna theory
Crystallography
Fast algorithms
Gabor theory and
applications
Image processing
Number theory
Optics

Partial differential
equations
Prediction theory
Radar applications
Sampling theory
Spectral estimation
Speech processing
Stochastic processes

Time-frequency analysis
Time series
Tomography
Turbulence
Uncertainty principles
Wavelet theory and
applications

Editorial Statement

The Journal of Fourier Analysis and Applications is a journal of the mathematical sciences devoted to Fourier analysis and its applications. The subject of Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. The usefulness and influence of Fourier analysis continues to grow, both in the number of topics it affects, and in the significance of the ideas and techniques it brings to bear on these topics. Fourier analysis is a deep subject with its own identity; and it is a subject whose relationship with other disciplines yields a multifaceted personality.

Historically, Fourier series were developed in the analysis of some of the classical partial differential equations (PDE) of mathematical physics; and these series were used to solve such equations. In order to understand Fourier series and what sorts of solutions they could represent, some of the basic notions of analysis were defined, e.g., the concept of function. Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series; and Cantor's set theory was developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of the problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is not only the natural setting for important parts of PDE, but for many other problems in engineering, mathematics, and the sciences. For example, Wiener's Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDE has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the Fast Fourier Transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency and time-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory.

This is a remarkable period since many engineers, mathematicians, and scientists are really talking (and listening) seriously to one another in areas such as speech and image processing, turbulence, PDE, numerical analysis, etc. Fourier analysis seems to be an endless well of inspiration in certain of these topics, and is making new inroads in others, especially with recent theoretical and algorithmic developments in wavelet theory.

We envisage a journal which draws upon the aforementioned Fourier technologies, and which responds creatively to significant engineering, mathematics, and scientific problems.

John J. Benedetto
Founding Editor-in-Chief