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Alexey N. Karapetyants • Vladislav V. Kravchenko • Elijah Liflyand • Helmuth R. Malonek Editors

# Operator Theory and Harmonic Analysis

OTHA 2020, Part I – New General Trends and Advances of the Theory



**Editors** 

Alexey N. Karapetyants Institute of Mathematics, Mechanic and Computer Sciences and Regional Mathematical Center Southern Federal University Rostov-on-Don, Russia

Elijah Liflyand Department of Mathematics Bar-Ilan University Ramat-Gan, Israel Vladislav V. Kravchenko Department of Mathematics Cinvestav Santiago de Queretaro, Querétaro, Mexico

Helmuth R. Malonek Department of Mathematics University of Aveiro Aveiro, Portugal

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#### **Preface**

This is the first volume of the two-volume series entitled

Operator Theory and Harmonic Analysis.

Vol. 1: New General Trends and Advances of the Theory

and

Vol. 2: Probability-Analytical Models, Methods, and Applications

Volume 1 is devoted to harmonic analysis and its applications in general, while Volume 2 is focused on probabilistic and mathematical (statistical) methods in applied sciences, but still in the context of general harmonic analysis and its numerous applications.

The volumes' readership is the pool of researchers interested in various aspects of harmonic analysis and operator theory: real and complex variable methods, applications to PDE's, mathematical modeling based on applied harmonic analysis and probability-analytical methods, and exploration of new themes and trends.

The contributions to both volumes are based on the matter supposed to be presented at the Annual International Scientific Conference on Modern Methods and Problems of Operator Theory and Harmonic Analysis and Their Applications (OTHA-2020, http://otha.sfedu.ru/), canceled due to Covid19 restrictions.

The Editors are very grateful to all the authors for their valuable contributions and for a strong willingness to support mathematical activities and communications in the hope of the soonest resumption of regular conferences and safe mutual visits. The Editors express an immense sorrow on the occasion of the recent loss of remarkable scientists and brilliant persons, Hrachik Hayrapetyan (Armenia), who is one of the authors of the first volume, Vladimir Pilidi (Russia), who was an active member of Program Committees of OTHA conferences, and Vladimir Nogin (Russia), who was a colleague and a teacher of quite a few participants of OTHA.

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The first volume contains words in memoriam of our dear friends Hrachik Hayrapetyan, Vladimir Pilidi, and Vladimir Nogin.

E. Liflyand

H. Malonek

Rostov-on-Don, Russia A. Karapetyants Santiago de Queretaro, Mexico V. Kravchenko Ramat-Gan, Israel Aveiro, Portugal

### In Memory of Prof. Hrachik M. Hayrapetyan (25.10.1946–06.11.2020)



On November 6, 2020, the world mathematical community lost a brilliant mathematician and a wonderful personality Hrachik Hayrapetyan.

Professor Hayrapetyan was born in 1946 in Dilijan (Armenia). His mathematical talents were noticed since his adolescence. His mathematical inclinations were influenced by contacts with his first teacher A. Sahakyan. In 1964, he started his studies at the Faculty of Mathematics and Mechanics of the Yerevan State University from which he graduated in 1969. After two years of service in the Soviet Army, he entered the Institute of Mathematics of the National Academy of Sciences of Armenia as a junior scientific researcher. Since then, his collaboration with the academician Mkhitar Jrbashyan started, who proposed to him the study of free interpolation and basis properties of rational fractions. Hrachik Hayrapetyan succeeded in discovering a series of essential results in this research field. In particular, he proved that if the multiplicities in the interpolation problem are not bounded, then this problem may have no solution and the rational fractions may fail to be a basis in the closure of their linear span. In 1975, he completed his PhD thesis. In 1979, he entered the National Polytechnic University of Armenia as an associate professor of the Chair of Applied Mathematics. At that time, the scientific group of Prof. N.E. Tovmasyan was developing the theory of boundary value problems for partial differential equations. H.M. Hayrapetyan was actively involved in this research. As a specialist in the theory of complex variable functions, he

was interested in theoretical-functional approach to these problems. He succeeded in obtaining a series of important results. Particularly, it is worth mentioning the new formulation of the classical Riemann boundary value problem, which allowed to solve this problem first in the classes of integrable functions and afterwards in the class of essentially bounded functions. Later, applying proposed method, Prof. Hayrapetyan with his students investigated boundary value problems in various functional spaces. The obtained results not only extended the theory of boundary value problems but also permitted to develop the theory of elliptic partial differential equations. Hrachik Hayrapetyan defended his Doctor of Science thesis "Riemann-Hilbert boundary value problem in the sense of mean convergence and applications in the theory of elliptic partial differential equations" in the M.V. Lomonosov Moscow State University. Specialists evaluated his results as a great success in the theory of boundary value problems. Developing his theory during the last decade, he studied boundary value problems in weighted classes of functions. He succeeded to describe the classes of functions, where the Dirichlet and Riemann-Hilbert boundary value problems in the classes of polyanalytic and polyharmonic functions are normally solvable in both bounded and unbounded domains. These results are highly evaluated by specialists in Armenia as well as abroad.

Hrachik Hayrapetyan was one of the members of the first elected Council of the Armenian Mathematical Union created in 1991 following Armenia's independence from the Soviet Union.

Prof. Hayrapetyan was an active organizer. He served two terms as the President of Mathematical Association of Armenia and he was the Head of specialized mathematical education chair in National Polytechnic University of Armenia and the Head of Mathematical Analysis and Function Theory chair in the Yerevan State University. His devotion to the science and excellent empathy skills helped him to interest young students in mathematical research. He was a scientific advisor of 15 PhD theses; his students continue the work on ideas of their teacher and mentor, Hrachik Hayrapetyan, in various universities and research institutions of Armenia.

The fond memory of our friend will forever rest in our hearts.

Doctor of Science, Professor Armenak H. Babayan Doctor of Science, Professor Vanya A. Mirzoyan Doctor of Science, Professor Levon Z. Gevorgyan

Chair of Specialized Mathematical Education of National Polytechnic University of Armenia

#### In Memory of Prof. Vladimir Pilidi (07.11.1946–19.01.2021)



Vladimir Pilidi became a student at the Rostov State University (now Southern Federal University) in 1964. All his scientific career from a talented student (diploma with honors) to distinguished Chair was related to this university during 57 years.

In 1972, under the guidance of Professor I.B. Simonenko, he defended his Ph.D. thesis "Local method for the study of linear operator equations of the type of bisingular integral equations," and in 1990 at the Dissertation Council of the Tbilisi Institute of Mathematics named after I. A. Razmadze of the Georgian Academy of Sciences, he defended doctoral dissertation (second degree) "Bisingular operators and classes of operators close to them", which became a significant scientific achievement that enriched the general theory of operators of local type. Seven students of Vladimir Pilidi became candidates of science (PhD's).

Professor Pilidi was a highly qualified expert in the field of mathematics and its applications, reviewer of scientific articles, member and chairman of the Dissertation Council, scientific consultant of several research institutions, chairman of the State Examination Commissions of universities. After he became the head of the Chair of Informatics and Computational Experiment in 2000, Vladimir Pilidi expanded his area of scientific interests towards the application of mathematical methods in cryptography, the theory of pattern recognition, and graphic information processing.

Professor Pilidi is known as the author of the bilocal method, an analogue of the classical local method of Simonenko, and in his research he successfully applied it to the study of bisingular and related operators, as well as algebras of such operators. Thanks to these achievements, the name of Vladimir Pilidi will forever remain among the names of outstanding researchers in analysis and operator theory.

Professor Pilidi actively participated in the scientific life of the Mathematics Department, in the organization of scientific seminars, conferences, and schools. He was one of the main organizers of the OTHA conference series, a regular participant, and a member of the Program Committee of these conferences. Professor Pilidi and his students made a valuable contribution to the development of this series of conferences and to the development of publication activity following the conferences.

Vladimir Pilidi is known as a brilliant lecturer of various courses in mathematics and computer science. He had remarkable achievements as a teacher in the lecture course in algebra and geometry for students of applied and mechanical engineering, which he taught for about 20 years. His textbook "Linear Algebra" (Vuzovskaya Kniga, Moscow, 2005), co-authored with A.V. Kozak, is standard for other authors. Other textbooks by Professor Pilidi are: "Mathematical Analysis" (Phoenix, 2009), "Mathematical Foundations of Information Security" (Southern Federal University, 2019), and Analytic Geometry (Southern Federal University, 2020). Vladimir Pilidi developed a deep modern course on mathematical methods of cryptography, which he taught to students of the Department of Fundamental Informatics and Information Technology and students of the Department of Applied Mathematics, specialized in the field of mathematical methods of information security.

His distinguished features were not only erudition and professionalism but also modesty, discreetness, and goodwill in relations with colleagues and students. Vladimir Pilidi was a wonderful head of his mathematical family. His wife, daughter, and son-in-law devoted themselves to mathematics, and his grandchildren are preparing to become mathematicians as well.

The bright memory of Vladimir Pilidi—of a mathematician, a teacher, and a brilliant person, will remain in our hearts.

I. M. Erusalimskiy A. N. Karapetyants V. S. Rabinovich S. G. Samko

#### In Memory of Vladimir Nogin (20.12.1955–31.05.2021)



Dr. Nogin Vladimir Alexandrovich was born on December 20, 1955. Vladimir Nogin graduated from the Faculty of Mechanics and Mathematics of the Rostov State University (now it is Southern Federal University) in 1979, defended his Ph.D. thesis in 1982 and worked 35 years as an assistant professor, senior teacher and then associate professor of the Department (Chair) of Differential and Integral Equations at the same University. During his work at the university, V.A. Nogin taught courses in mathematical analysis, higher mathematics, and mathematical physics. He also developed and taught more than five special courses for undergraduate and postgraduate math students, which included contemporary results in the field of functional analysis and mathematical physics.

Dr. Nogin's scientific interests were in the classical area of analysis related to the study of operators of mathematical physics, the construction and study of fractional powers of these operators, their inversion, and the description of the image of such operators in the framework of Lebesgue spaces. At the same time, he dealt with questions of functional analysis—the description of function spaces that arise in analysis in the context of the above-mentioned theory of operators. He and his students obtained profound results in this theory; he successfully developed the so-called method of approximate inverse operators. He has published about 70 scientific papers and a significant number of textbooks.

Vladimir Alexandrovich always found enough time for his students, and scientific work was his main passion in life. 8 PhD theses defended under his supervisorship. One of his students, Mikhail Gurov, became the teacher of the year in the Russian Federation in 2020.

Vladimir Alexandrovich was distinguished by his modesty and delicacy in relation to colleagues. The bright memory of Dr. V.A. Nogin will always be in the hearts of his colleagues and students.

On behalf of the colleagues and students,

O. G. Avsyankin, A. P. Chegolin, M. N. Gurov, A. N. Karapetyants, D. N. Karasev, B. G. Vakulov

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