

**Springer Proceedings in Mathematics &
Statistics**

Volume 291

Springer Proceedings in Mathematics & Statistics

This book series features volumes composed of selected contributions from workshops and conferences in all areas of current research in mathematics and statistics, including operation research and optimization. In addition to an overall evaluation of the interest, scientific quality, and timeliness of each proposal at the hands of the publisher, individual contributions are all refereed to the high quality standards of leading journals in the field. Thus, this series provides the research community with well-edited, authoritative reports on developments in the most exciting areas of mathematical and statistical research today.

More information about this series at <http://www.springer.com/series/10533>

Alexey Karapetyants · Vladislav Kravchenko ·
Elijah Lifyand
Editors

Modern Methods in Operator Theory and Harmonic Analysis

OTHA 2018, Rostov-on-Don, Russia,
April 22–27
Selected, Revised and Extended Contributions

Editors

Alexey Karapetyants
Institute of Mathematics, Mechanics and
Computer Sciences and Regional
Mathematical Center
Southern Federal University
Rostov-on-Don, Russia

Vladislav Kravchenko
Department of Mathematics
Cinvestav
Santiago de Querétaro, Querétaro, Mexico

Elijah Liflyand
Department of Mathematics
Bar-Ilan University
Ramat Gan, Israel

ISSN 2194-1009 ISSN 2194-1017 (electronic)
Springer Proceedings in Mathematics & Statistics
ISBN 978-3-030-26747-6 ISBN 978-3-030-26748-3 (eBook)
<https://doi.org/10.1007/978-3-030-26748-3>

Mathematics Subject Classification (2010): 47Bxx, 46Exx, 35Qxx, 42-xx, 46E15, 46E30, 46E35, 42-00

© Springer Nature Switzerland AG 2019

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This volume assembles new results concerning the methods, problems, and applications of operator theory and harmonic analysis. Presented by leading researchers, chapters notably touch on recent developments in spectral theory, with applications to PDEs and real-world problems, along with a deep emphasis on the new trends in operator theory and function spaces theory, such as variable exponent spaces and objects of variable nature. The published results will mainly attract researchers who are interested in real and complex variable methods, applications to PDEs, and exploration of new themes and trends in harmonic analysis and operator theory. The relevance of the topics covered in this volume, is stipulated, in particular, by the study of complicated multiparameter objects that require the involvement of operators with variable parameters and functional spaces with fractional and even variable exponents.

This volume consists mainly of the works of speakers at the annual *International Scientific Conference of Modern Methods and Problems of Operator Theory and Harmonic Analysis and Their Applications (OTHA-2018)*, which took place on April 22–27, 2018, at the Southern Federal University in Rostov-on-Don, Russia. Information on this and preceding conferences is available at the conference website <http://otha.sfedu.ru/>.

Every year numerous investigators from research and educational institutions worldwide participate in OTHA conferences. Among them are mathematicians from Russia, China, the United States, the United Kingdom, Israel, Finland, Germany, Canada, Italy, Spain, Portugal, Sweden, Mexico, Colombia, Armenia, Azerbaijan, Jordan, Iran, UAE, Belarus, Kazakhstan, Poland, Lithuania, Uzbekistan, and Lithuania. One of the main features of OTHA conferences is the existence of a very special creative atmosphere. This results from both opportunities to form numerous contacts and collaborations and the sharing of inspirational presentations and ideas. The conference in 2018 was supported by the Regional Mathematical Center of the Southern Federal University in collaboration with the Institute of Mathematics, Mechanics and Computer Science of the Southern Federal University. It was also supported by a grant from the Russian Foundation for Basic

Research (<https://www.rfbr.ru/>) and by the International Society for Analysis and its Applications (ISAAC, isaacmath.org).

We also note that this volume is prepared as part of a special agreement between the ISAAC society and Springer. We thank the ISAAC society for its many years of fruitful collaboration and support, and we thank Springer for providing such a great opportunity. We thank all the authors of this volume for their participation and contributions.

We express our immense regret at the loss of two participants (plenary speakers) who frequently presented at OTHA conferences, our dear friends and great mathematicians, Stasys Rutkauskas (Lithuania) and Garnik Karapetyan (Armenia), both passing in 2018. We have reserved several pages of this volume to honor the memory of these great researchers and friends.

Rostov-on-Don, Russia
Santiago de Querétaro, Mexico
Ramat Gan, Israel

Alexey Karapetyants
Vladislav Kravchenko
Elijah Lifyand

Professor Garnik Karapetyan (1958/02/03–2018/11/29)



With kind permission of Heghine Petrosyan. Copyright remains with the author

Professor Garnik Karapetyan, an outstanding mathematician, a specialist in differential equations and differentiable functions of several variables and other related areas, and a Distinguished Professor of the Russian–Armenian University (RAU), passed away on November 29, 2018.

The diverse scientific activity of Prof. G. Karapetyan meant that he was well-known in the mathematical community. One of the main parts of the theory of differential equations and functional analysis, the general theory of differential equations in the spaces of differentiable functions, is associated with his name. His research played a significant role in the development of a general theory of integral representations and embedding theorems in multianisotropic functional spaces.

Professor Garnik Karapetyan was born on February 3, 1958, in Leninakan (now Gyumri), Armenian SSR. In 1975, he started undergraduate studies at the Faculty of Mechanics and Mathematics of Yerevan State University (YSU) and graduated *cum laude* in 1980. In 1983 he defended his Ph.D. thesis and started work as an assistant at the Department of Numerical Analysis of the Faculty of Informatics and Applied Mathematics at YSU. From 1990 to 1994, he held the position of dean of the

Faculty of Informatics and Applied Mathematics at YSU. He was head of the Mathematics Olympiad Committee of Armenia for 2 years starting in 1997. In 2002, he founded the Department of Mathematics and Mathematical Modeling at RAU. In 2007, he successfully defended his D.Sc. thesis entitled “Properties of the solutions of regular hypoelliptic equations.” In 2008, he received the academic rank of professor from the Higher Attestation Commission of the Republic of Armenia and the title of Distinguished Professor at RAU. In 2008, he became a member of the Scientific Council of RAU and the Dissertation Council in Mathematics of the Republic of Armenia. From 2015, he was director of the Applied Mathematics and Informatics program at RAU. He was the first director of the Institute of Mathematics and Informatics at RAU. Professor Garnik Karapetyan was awarded the Medal of Anania Shirakatsi, by decree of the President of the Republic of Armenia.

For many years, he was co-head of the scientific seminar “Functional spaces and their applications in mathematical physics,” first at YSU and then RAU. He authored and co-authored 8 textbooks and manuals and more than 60 research articles. His research was mainly devoted to elliptic and hypoelliptic differential equations, the study of the properties of functions in different multianisotropic spaces, and the integral representations and embedding theorems for functions in multianisotropic spaces.

The size limitation applied to this text makes it possible to only briefly describe some of, but not all, the studies he conducted, often with students, which resulted in significant advances and a new deep understanding. His main research activity encompassed:

- The convergence of Galerkin approximations to the solution of the Dirichlet problem for different classes of hypoelliptic equations.
- Liouville and Phragmén–Lindelöf type theorems for general regular equations.
- The behavior of solutions of a certain class of hypoelliptic and μ -elliptic equations in bounded and unbounded domains.
- Estimates for solutions of quasi-elliptic equations in Hölder spaces.
- Schauder-type estimates for solutions of semi-elliptic equations with variable coefficients.
- The study of solutions of hypoelliptic and degenerate equations in a half-space.
- The study of boundary value problems for regular quasilinear equations in unbounded domains.
- Proof of stabilization of solutions for regular and hypoelliptic equations of different classes.
- The study of solutions of nondegenerate equations depending on a parameter.
- The study of eigenvalues of self-adjoint semi-elliptic operators.
- Proof of multiplicative inequalities of Gagliardo–Nirenberg type in multianisotropic Sobolev spaces.
- Approximations of solutions of semi-elliptic equations in a half-space.
- Computation of the index of semi-elliptic operators.
- Proof of the Noetherianness of semi-elliptic and nondegenerate operators.

- Application of the small parameter method for the approximation of eigenvalues of regular hypoelliptic operators.
- Integral representations of functions in different multianisotropic spaces and their embedding theorems.

Professor Garnik Karapetyan was a highly qualified professional in his field and a talented leader. One of his distinguishing qualities was paying particular attention and respect to people and their problems. He was impeccably honest and principled in his work and tactful in relationships with his family members, relatives, friends, colleagues, and students. He was tremendously charismatic; he enjoyed exceptional trust and reverence from students. Not only did he demonstrate genuine interest in everything new in science and life in general, but he also triggered interest in his students and those surrounding him.

Professor G. Karapetyan was not only well-known among scientists and academics; the general public also knew him thanks to the educational program “Open Lesson,” broadcast on public TV in Armenia.

What is particularly sad for all his friends and colleagues is that he left this world during his creative ascent. During his last 5–6 years he actively conducted fruitful research on the integral representations of functions in multianisotropic Sobolev spaces and the embedding theorems for such spaces. His most recent work, in the field of isotropic or anisotropic Sobolev spaces, formed part of a body of work that included research by distinguished scientists including S. Sobolev, L. Schwartz, S. Nikolskii, S. Agmon, O. Besov, H. Triebel, E. Gagliardo, P. Lizorkin, I. Slobodetsky, I. Nechas, V. Burenkov, S. Uspenskii, V. Solonnikov, G. Demidenko, A. Kufner, and many others.

Most recently, Prof. G. Karapetyan obtained integral representations for functions in multianisotropic Sobolev spaces generated by regular Newton polyhedra, and, based on these representations, proved embedding theorems for such spaces. Some of the results he obtained are not yet published. His plans for future work included the description of traces of functions in multianisotropic spaces on hypersurfaces of different dimensions and theorems on the continuation of functions outside their domain of definition. We hope that this work will be continued by his disciples.

The family, relatives, friends, colleagues, and students of Garnik Karapetyan, as well as Armenian mathematics, suffered an irretrievable loss due to his untimely death.

Cherished memories of Garnik Karapetyan will forever remain in our hearts.

Adamyanyan G. V., Aramyanyan R. H., Avetisyan P. S., Babayan A. H., Baghdasaryan A. G., Berberyan S. L., Darbinyan A. A., Darbinyan A. R., Gevorgyan G. G., Ghazaryan E. M., Ghazaryan H. G., Harutyunyan K. V., Harutyunyan T. N., Margaryan V. N., Petrosyan H. A., Sandoyan E. M., Sargsyan G. Z., Tonoyan R. N., and Tumanyan A. G.

Professor Stasys Rutkauskas **(1951/06/06–2018/10/29)**



With kind permission of Gintautas Dzemyda. Copyright remains with the author

Professor Stasys Rutkauskas was born on June 6, 1951, in Nečionių in the Širvintai region of Lithuania. He graduated from secondary school in 1968 in Širvintai. In 1973 he finished his studies at the Faculty of Mathematics of Vilnius University (VU) and acquired a diploma as a mathematician. From 1974 to 1977 he was a Ph.D. student at the Institute of Mathematics and Informatics in Vilnius, and in 1978 he defended his Ph.D. theses at Minsk University, receiving a doctoral degree in mathematics. From 1977 to 1980 he worked as a junior research fellow. Starting in 1980 he worked as a senior research fellow at the Institute of Mathematics and Informatics. From 1995 to 2005 he was the Scientific Secretary of the Institute, and in 2005, after defending his habilitation theses and gaining a D.Sc., he became vice-director for research at the Institute of Mathematics and Informatics (nowadays Vilnius University Institute of Data Science and Digital Technologies).

Professor S. Rutkauskas had more than 50 scientific papers published and presented many invited talks at international scientific conferences. The topics of his research included boundary value problems for degenerate elliptic systems and the asymptotic behavior of their solutions near singular points. He actively collaborated

with mathematicians from other countries while working on these problems, delivering special lecture courses at foreign universities on multiple occasions.

Professor S. Rutkauskas was a member of the editorial boards of several international mathematical journals including the *Lithuanian Mathematical Journal* and *Mathematical Modelling and Analysis*. He gave lectures to bachelor and master's students in VU, Vilnius Pedagogical University (VPU), and Vilnius Gediminas Technical University (VGTU). His textbook for Ph.D. students entitled *Asymptotic Methods for Ordinary Differential Equations* proved very helpful to young mathematicians. He was the leader of the scientific project entitled "Informatics and Mathematics doctoral studies development (InMaDra)," supported by European structural funds.

He loved his family very much. He had two sons and three granddaughters. He was warm, loved by all, a modest man who left unfinished both works and ideas.

Prof. Dr. Habil. Gintautas Dzemyda
Full member of the Lithuanian Academy of Sciences
Director of the Institute of Mathematics and Informatics
Head of the Systems Analysis Department

Contents

Function Theory and Approximation Theory	
Some General Properties of Operators in Morrey-Type Spaces	3
Mikhail L. Goldman and Elza Bakhtigareeva	
Characterization of Parabolic Fractional Maximal Function and Its Commutators in Orlicz Spaces	35
Vagif S. Guliyev, Ahmet Eroglu and Gulnara A. Abasova	
Finite Trees Inside Thin Subsets of \mathbb{R}^d	51
A. Iosevich and K. Taylor	
Boundedness of Projection Operator in Generalized Holomorphic and Harmonic Spaces of Functions of Hölder Type	57
Alexey Karapetyants and Joel E. Restrepo	
Generalized Fourier Series by the Double Trigonometric System	67
K. S. Kazarian	
Hardy Type Inequalities in the Category of Hausdorff Operators	81
Elijah Lifyand	
Harmonic Analysis and Hypercomplex Function Theory in Co-dimension One	93
Helmuth R. Malonek, Isabel Cação, M. Irene Falcão and Graça Tomaz	
Paraproduct in Besov–Morrey Spaces	117
Yoshihiro Sawano	
Functional Analysis and Operator Theory	
Analogs of the Khintchin—Kolmogorov Inequalities in Discrete Morrey Spaces	137
Evgenii I. Bereznoi	

Mellin Convolution Equations	153
Roland Duduchava	
Integral Operators of the \mathcal{L}-Convolution Type in the Case of a Reflectionless Potential	175
Davresh Hasanyan, Armen Kamalyan, Martin Karakhanyan and Ilya M. Spitkovsky	
Spectral Theory for Nonlinear Operators: Quadratic Case	199
Yakov Krasnov	
Factorization of Order Bounded Disjointness Preserving Multilinear Operators	217
Anatoly G. Kusraev and Zalina A. Kusraeva	
Robbins–Monro Conditions for Persistent Exploration Learning Strategies	237
Dmitry B. Rokhlin	
On Widths of Invariant Sets	249
Ekaterina Shulman	
The Distance Function and Boundedness of Diameters of the Nearest Elements	263
Igor' G. Tsar'kov	
Differential Equations and Mathematical Physics	
The Influence of Oscillations on Energy Estimates for Damped Wave Models with Time-dependent Propagation Speed and Dissipation	275
Halit Sevki Aslan and Michael Reissig	
On a Dirichlet Problem for One Improperly Elliptic Equation	317
Armenak H. Babayan and Seyran H. Abelyan	
On the 1-dim Defocusing NLS Equation with Non-vanishing Initial Data at Infinity	337
Nikolaos Gialelis and Ioannis G. Stratis	
On Time-Global Solutions of SDE Having Nowhere Vanishing Initial Densities	363
Yuri E. Gliklikh	
On Transmutation Operators and Neumann Series of Bessel Functions Representations for Solutions of Linear Higher Order Differential Equations	371
Flor A. Gómez and Vladislav V. Kravchenko	
On a Boundary Value Problem with Infinite Index	387
H. M. Hayrapetyan	

**A Numerical Realization of the Wiener–Hopf Method
for the Kolmogorov Backward Equation** 399
Oleg Kudryavtsev and Vasily Rodochenko

On Waves Processes in Transversally-Inhomogeneous Waveguides 427
Alexander Vatulyan and Victor Yurov

Inverse Spectral Problems for Differential Systems 451
Vjacheslav Anatoljevich Yurko